NUMERICAL METHODS FOR WIRE STRUCTURES

by

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INTRODUCTION

MiniNec is a personal computer version of the mainframe numerical electromagnetics code (NEC) summarized in Tech. Doc. 516 of the Naval Ocean Systems Center [Julian, 1982]. The MiniNec code has been revised since its original development by the originators [Rockway, 1988] and several other authors [ACES, 1989]. The developers wrote their original version in BASIC for a small machine. The original was also converted for use with IBM BASIC and Fortran. The code worked for most problems, but the source code was extremely hard to follow (partly due to the version of BASIC originally used) and had a few philosophical difficulties, which caused errors for certain configurations. Some of difficulties have been addressed with more recent versions of MiniNec, typically incorporating a thin-wire approximation. However, the orientation towards BASIC and some of the original numerical methods still exists.

This paper redevelops the theory with an emphasis on the philophy and the necessary modifications which correct the previous problems. In addition, the associated software is written in Turbo Pascal[1] which lends itself to self documentation and a much easier understanding of the specific code. It is hoped that this presentation will help those who have had problems understanding and using the original code and documention. In addition, this provides development of the appropriate equations with an eye on the computational difficulties which arose in the original. (The executable code is available at no cost on bulletin boards as "ANTENNA.EXE" or from this author for a self-addressed, stamped envelope and diskette under DOS. Included with this program are two of the modified array plotting programs from the book by Stutzman and Thiele [1981].)

EQUATION BASICS

The starting point for electromagnetic (EM) interaction with thin, perfectly-electric conducting structures (PECs) [Davis, 1974] is the electric field integral equation (EFIE)

$$0 = \hat{\boldsymbol{n}}(\boldsymbol{r}) \times \frac{1}{\mathrm{j}\, k\, \eta} \left[k^2 \overline{\boldsymbol{A}} + \nabla \left(\nabla \cdot \overline{\boldsymbol{A}}\right)\right] \tag{1}$$

where

$$\overline{\boldsymbol{A}}(\boldsymbol{r}) = \mu_s \int_S \overline{\boldsymbol{J}}_s(\boldsymbol{r}')\, \Phi(\boldsymbol{r}, \boldsymbol{r}')\, \mathrm{d}s', \tag{2}$$

$$\Phi(\boldsymbol{r}, \boldsymbol{r}') = \frac{\mathrm{e}^{-\mathrm{j}kR}}{4\,\pi\, R}, \quad R = |\boldsymbol{r} - \boldsymbol{r}'\,|\,, \tag{3}$$

and k is the wave number given by $\omega\sqrt{\mu\epsilon}$. The latter quantity Φ is called the free space Green's function. It is common to separate part of the equation out as the incident field term to give

$$-\,\mathrm{j}\omega\mu\epsilon\; \hat{\boldsymbol{n}} \times \overline{\boldsymbol{E}}_{inc} = \hat{\boldsymbol{n}} \times [\, k^2\, \overline{\boldsymbol{A}} + \nabla\, (\, \nabla \cdot \overline{\boldsymbol{A}}\,)]. \tag{4}$$

The problem of determining $\overline{\boldsymbol{E}}_{inc}$ for a driven antenna will be considered later.

For wire antennas, we generally restrict our consideration to uniform circumferential currents flowing along the wires. Currents on end caps are ignored in this development and the antenna is considered to be constructed of hollow tubing for which this current is an exact assumption. (End caps are generally found to not cause any significant difference in the solution, at least for relatively thin structures.) An error should be expected at wire junctions with regard to exact current location, but with minimal effect. Since we effectively consider the average current about a cylinder, we shall also consider only the average incident fields.

[1] A C++ version is under development.

In using the integral equations, it is found that the magnetic-field integral equation (HFIE) is not well suited to wires, thus the use of the EFIE here. The HFIE may be modified to work, but tends toward a version of the EFIE when so modified [Mittra, etal, 1973]. Thus we work with the following EFIE averaged about the wire circumference:

$$
\begin{aligned}
-\,\mathrm{j}\omega\mu\epsilon\,\hat{\boldsymbol{\ell}}\cdot\overline{\boldsymbol{E}}_i &= k^2\,\frac{1}{2\,\pi}\int\limits_{\phi}\hat{\boldsymbol{\ell}}\cdot\overline{\boldsymbol{A}}\,\mathrm{d}\phi+\frac{1}{2\,\pi}\int\limits_{\phi}\hat{\boldsymbol{\ell}}\cdot\nabla\left(\nabla\cdot\overline{\boldsymbol{A}}\right)\mathrm{d}\phi \\
&= \frac{\hat{\boldsymbol{\ell}}}{2\,\pi}\cdot\left[k^2\int\limits_{\phi}\overline{\boldsymbol{A}}\,\mathrm{d}\phi+\int\limits_{\phi}\nabla\left(\nabla\cdot\overline{\boldsymbol{A}}\right)\mathrm{d}\phi\right]
\end{aligned}
\tag{5}
$$

where

$$
\begin{aligned}
\overline{\boldsymbol{A}}(\boldsymbol{r}) &= \mu\int\limits_{\ell'}\overline{\boldsymbol{J}}_s(\ell')\int\limits_{\phi'}\Phi\;\mathrm{a}'\;\mathrm{d}\phi'\;\mathrm{d}\ell' \\
&= \mu\int\limits_{\ell'}\hat{\ell}'\;I(\ell')\left[\frac{1}{2\,\pi}\int\limits_{\phi'}\Phi\;\mathrm{d}\phi'\right]\mathrm{d}\ell',
\end{aligned}
\tag{6}
$$

the surface current being replaced by the total wire current $I(\ell')$. The average azimuthal incident field is given by

$$
\hat{\boldsymbol{\ell}}\cdot\overline{\boldsymbol{E}}_i=\frac{1}{2\,\pi}\,\hat{\boldsymbol{\ell}}\cdot\int\limits_{\phi}\overline{\boldsymbol{E}}_{inc}\,\mathrm{d}\phi.
$$

The first integral of (5) is simply written as

$$
\frac{1}{2\,\pi}\int\limits_{\phi}\overline{\boldsymbol{A}}\,\mathrm{d}\phi=\int\limits_{\ell'}\hat{\boldsymbol{\ell}}'\;I(\ell')\left[\frac{\mu}{(2\,\pi)^2}\int\limits_{\phi}\int\limits_{\phi'}\Phi\;\mathrm{d}\phi'\;\mathrm{d}\phi\right]\mathrm{d}\ell'.
\tag{7}
$$

We may expresss the second term $\left(\nabla\cdot\overline{\boldsymbol{A}}\right)$ in terms of a Cauchy principal value integral as

$$
\nabla\cdot\overline{\boldsymbol{A}}=\fint\limits_{\ell'}I(\ell')\,\hat{\boldsymbol{\ell}}'\cdot\nabla\left[\frac{\mu}{2\,\pi}\int\limits_{\phi'}\Phi\;\mathrm{d}\phi'\right]\mathrm{d}\ell'
$$

and using integration by parts obtain

$$
\nabla\cdot\overline{\boldsymbol{A}}=\int\limits_{\ell'}\frac{\partial I}{\partial\ell'}(\ell')\left[\frac{\mu}{2\,\pi}\int\limits_{\phi'}\Phi\;\mathrm{d}\phi'\right]\mathrm{d}\ell'.
\tag{8}
$$

The second term in (5) now becomes

$$
\begin{aligned}
\frac{1}{2\,\pi}\;\hat{\boldsymbol{\ell}}\cdot\nabla\int\limits_{\phi}\nabla\cdot\overline{\boldsymbol{A}}\,\mathrm{d}\phi &= \frac{\partial}{\partial\ell}\int\limits_{\ell'}\frac{\partial I(\ell')}{\partial\ell'}\left[\frac{\mu}{(2\,\pi)^2}\int\limits_{\phi}\int\limits_{\phi'}\Phi\;\mathrm{d}\phi'\;\mathrm{d}\phi\right]\mathrm{d}\ell' \\
&= \frac{\partial}{\partial\ell}\int\limits_{\ell'}\frac{\partial I(\ell')}{\partial\ell'}\;[\mu\,K(\ell,\ell')]\;\mathrm{d}\ell',
\end{aligned}
\tag{9}
$$

where K has been introduced to represent the double integral over ϕ and ϕ'. The end result is

$$-\,\mathrm{j}\omega\epsilon\,\hat{\boldsymbol{\ell}}\cdot\overline{\boldsymbol{E}}_{\mathrm{i}} = \int_{\ell'} \left(\hat{\boldsymbol{\ell}}\cdot\hat{\boldsymbol{\ell}}'\right) \mathrm{k}^{2}\, I(\ell')\, K(\ell,\ell')\, \mathrm{d}\ell' + \frac{\partial}{\partial\ell}\int_{\ell'} \frac{\partial I(\ell')}{\partial\ell'}\, K(\ell,\ell')\, \mathrm{d}\ell' \tag{10}$$

where

$$\hat{\boldsymbol{\ell}}\cdot\overline{\boldsymbol{E}}_{\mathrm{i}} = \hat{\boldsymbol{\ell}}\cdot\int_{\phi}\overline{\boldsymbol{E}}_{\mathrm{inc}}\,\mathrm{d}\phi \tag{11}$$

and

$$K(\ell,\ell') = \frac{1}{(2\,\pi)^{2}}\int_{\phi}\int_{\phi'}\Phi(\boldsymbol{r},\boldsymbol{r}')\,\mathrm{d}\phi'\,\mathrm{d}\phi. \tag{12}$$

APPROXIMATION OF KERNEL

The computation of the kernel $K(\ell,\ell')$ and the related integrals in ℓ' are typically the most difficult problem of the numerical development for wire antenna analysis. In this section we will consider this computation and an integral of K over ℓ' for the structure shown in Fig. 1. For this purpose, we write K as

$$K(\ell,\ell') = \frac{1}{16\,\pi^{3}}\int_{\phi}\int_{\phi'}\frac{\mathrm{e}^{-jkR}}{R}\,\mathrm{d}\phi'\,\mathrm{d}\phi \tag{13}$$

where $R = |\,\boldsymbol{r}-\boldsymbol{r}'\,|$. Let us define several terms for Fig. 1 as follows:

$a =$ radius at $\boldsymbol{r}$,
$a' =$ radius at $\boldsymbol{r}'$,
$\boldsymbol{r}_0 =$ axial position of $\boldsymbol{r}$,
$\boldsymbol{r}_s =$ axial position of $\boldsymbol{r}'$,
$\rho = \perp$ distance of $\boldsymbol{r}_0$ from source wire axis,

and

$\rho' = \perp$ distance of $\boldsymbol{r}_s$ from observation wire axis.

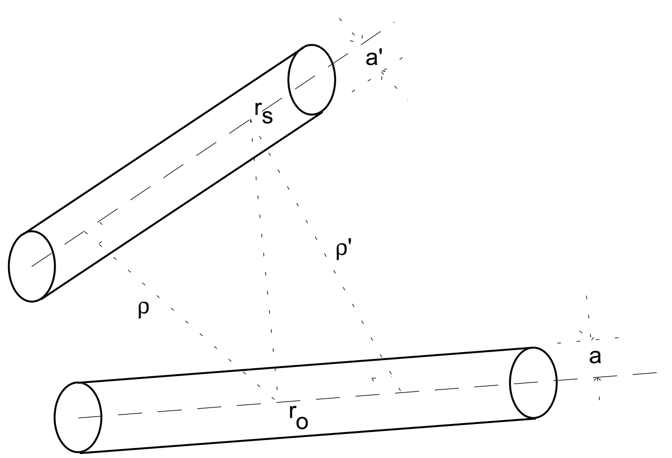

Figure 1. Coupled wire geometry.

Then we may define the distance R by

$$R^2 = R_0^2 + (a+a')^2 - 4\,aa'\,\cos^2\left(\frac{\phi-\phi'}{2}\right) + 2\left[a'\rho\cos(\phi'-\phi_s) + a\rho'\,\cos(\phi-\phi_o) + 2\,aa'\,\sin^2\frac{\theta}{2}\,\cos\phi'\,\cos\phi\right] \tag{14}$$

where θ is the included angle between the positive directions of the wires and ϕ, ϕ' are the angles around the wires as already suggested. The angles ϕ_o and ϕ_s provide the proper orientation for the ρ and ρ' projections. For ρ and ρ' equal to zero, this is the standard form for the straight wire structure with a possible variation in radius. The distance R_0 is the distance between axial positions and dominates the equation if ρ or ρ' become much larger than the radii. For this reason and simplicity of computation, the distance R will be approximated by

$$R^2 = R_0^2 + (a+a')^2 - 4\,aa'\,\cos^2\left(\frac{\phi-\phi'}{2}\right) \tag{15}$$

in which we may replace $(\phi-\phi')/2$ with $(\alpha-\pi/2)$. With this latter substitution, K takes on the simpler form of

$$K(\ell,\ell') = \frac{1}{4\pi^2 R'}\int_{-\pi/2}^{\pi/2}\frac{\mathrm{e}^{-\mathrm{j}kR}}{\sqrt{1-\beta^2\sin^2\alpha}}\,\mathrm{d}\alpha. \tag{16}$$

The distance $R' = \sqrt{R_0^2 + (a+a')^2}$ and the variable β is given by

$$\beta^2 = \frac{4\,aa'}{R'^2}\ . \tag{17}$$

For large separations, we may simply integrate a thin wire approximation to the kernel. If we integrate a Taylor approximation to $1/R$, then a slight modification to the standard thin − wire approximation becomes apparent and we write

$$K(\ell,\ell') \simeq \frac{\mathrm{e}^{-\mathrm{j}kR_a}}{4\pi R_a}\ ,\ R_a = \sqrt{R_0^2 + a^2 + a'^2}\ . \tag{18}$$

When the region of the singularity ($R \simeq a,\ a'$) is approached, this form is not very accurate. Thus we must resort to a more accurate form requiring the extraction of the singular parts which may be integrated analytically.

To simply evaluate K near the singularity, we extract the essential singularity of the denominator by adding and subtracting unity to the numerator. Neglecting the exponential, the form of K is an elliptic integral of the first kind. Following these suggested steps, we obtain the following:

$$K(\ell,\ell') = \frac{1}{2\pi^2 R'}\int_0^{\pi/2}\frac{\mathrm{d}\alpha}{\sqrt{1-\beta^2\sin^2\alpha}} + \frac{1}{8\pi^2}\int_{-\pi}^{\pi}\frac{\mathrm{e}^{-\mathrm{j}kR}-1}{R}\,\mathrm{d}(\phi'-\phi). \tag{19}$$

Since the first term has the form of the elliptic integral of the first kind $\mathrm{F}\left(\frac{\pi}{2},\beta\right)$, we write K as

$$K(\ell,\ell') = \frac{\beta}{4\pi^2\sqrt{aa'}}\ \mathrm{F}\left(\frac{\pi}{2},\beta\right) + \frac{1}{8\pi^2}\int_{-\pi}^{\pi}\frac{\mathrm{e}^{-\mathrm{j}kR}-1}{R}\,\mathrm{d}(\phi'-\phi). \tag{20}$$

The second term may be evaluated numerically using R_a since no singularity exists in this residual. This residual may be written

$$K_R(\ell,\ell') = \frac{e^{-jkR_a} - 1}{4\,\pi\,R_a}\ . \tag{21}$$

The numerical evaluation of (21) requires care and leads to the use of a sinusoid in the numerator times a phase function. To make the form of (20) useful, we must expand the elliptic integral in order to retain the necessary accuracy for numerical computation. For computation, the elliptic integral is expanded in a series as

$$F\left(\frac{\pi}{2},\beta\right) = [a_0 + a_1\,m + a_2\,m^2 + a_3\,m^3 + a_4\,m^4]$$
$$- [b_0 + b_1\,m + b_2\,m^2 + b_3\,m^3 + b_4\,m^4]\ \ell\mathrm{n}\,(m) + \epsilon(\beta^2) \tag{22}$$

where $|\epsilon| < 2 \times 10^{-8}$ for $|\beta| \in (0,1)$ [Ambromowitz and Stegan, 1964]. The ϵ term is usually neglected and the essential singularity of Φ extracted for analytical treatment. The factor m is defined as $(1 - \beta^2)$ and is only a problem as β approaches unity ($\boldsymbol{r}_o$ approaches $\boldsymbol{r}_s$). The dominant terms a_0 and $b_0\ell\mathrm{n}(m)$ [0.5 ℓn(16) and 0.5 $\ell\mathrm{n}(m)$ respectively] in K are given by

$$\frac{1}{2\pi^2 R'}\ (a_0 - b_0\ \ell\mathrm{n}(m)) = \frac{1}{4\pi^2 R'}\ \ell\mathrm{n}\left[\frac{16\,(\,|\,\boldsymbol{r}_0 - \boldsymbol{r}_s\,|\,^2 + (a+a')^2)}{|\,\boldsymbol{r}_0 - \boldsymbol{r}_s\,|\,^2 + (a-a')^2}\right]. \tag{23}$$

The essence of this form is the logrithmic singularity if $a = a'$. This form may be separated further into a nonsingular part and an integrable part which may be integrated in closed form over a source wire. One such expansion may be written as

$$\frac{1}{2\pi^2 R'}\,[a_0 - b_0\ \ell\mathrm{n}(m)] = \frac{1}{4\pi^2\,\sqrt{\rho^2 + (a+a')^2}}\ \left[1 + \frac{1-\xi}{\xi}\right]$$
$$*\ \ell\mathrm{n}\left[\frac{16\,\left[(\ell-\ell')^2 + \rho^2 + (a+a')^2\right)]}{(\ell-\ell')^2 + \rho^2 + (a-a')^2}\right] \tag{24}$$

The term ξ is given by $\left[R' \,/\, \sqrt{\rho^2 + (a+a')^2}\right]$ which approaches unity about the singularity. Thus the terms in ξ may be treated numerically. The last term is integrable in ℓ' in closed form as

$$\int_{\ell'} \ell\mathrm{n}\left[\frac{16\,\left[(\ell-\ell')^2 + \rho^2 + (a+a')^2\right]}{(\ell-\ell')^2 + \rho^2 + (a-a')^2}\right] d\ell'$$
$$= \Big\{(\ell' - \ell)\ \ell\mathrm{n}\Big[16\,\left(R_0^2 + (a+a')^2\right)/\left(R_0^2 + (a-a')^2\right)\Big]$$
$$- 2\,\sqrt{\rho^2 + (a-a')^2}\ \tan^{-1}\left[\frac{\ell' - \ell}{\sqrt{\rho^2 + (a-a')^2}}\right]$$
$$- 2\,\sqrt{\rho^2 + (a+a')^2}\ \tan^{-1}\left[\frac{\ell' - \ell}{\sqrt{\rho^2 + (a+a')^2}}\right]\Big\}. \tag{25}$$

To summarize the approximation near the singularity we have

$$
\begin{aligned}
\mathrm{K}(\ell,\ell') \simeq\ & \frac{1}{4\pi^2 \sqrt{\rho^2 + (a+a')^2}}\ \ell\mathrm{n}\left[\frac{16\left[(\ell-\ell')^2+\rho^2+(a+a')^2\right]}{(\ell-\ell')^2+\rho^2+(a-a')^2}\right] \\
& + \left\{\left[(1-\xi)\, a_0 + a_1\, m + a_2\, m^2 + a_3\, m^3 + a_4\, m^4\right]\right. \\
& \left. - \left[(1-\xi)\, b_0 + b_1\, m + b_2\, m^2 + b_3\, m^3 + b_4\, m^4\right] \ell\mathrm{n}(m)\right\} \frac{1}{2\pi^2 R'} \\
& - j\frac{\sin\left(\frac{kR_a}{2}\right)}{2\,\pi\, R_a}\mathrm{e}^{-\mathrm{j}\frac{kR_a}{2}}
\end{aligned} \tag{26}
$$

where

$$
\begin{aligned}
R_0 &= \mid \boldsymbol{r} - \boldsymbol{r}' \mid , \\
(R')^2 &= R_0^2 + (a+a')^2, \\
\ell - \ell' &= \text{component of } (\boldsymbol{r}-\boldsymbol{r}') \text{ along source wire}, \\
r &= \perp \text{ distance from source wire to } \boldsymbol{r}, \\
R_a^2 &= R_0^2 + a^2 + a'^2, \\
\xi &= R' \,/\, \sqrt{\rho^2 + (a+a')^2}\ ,
\end{aligned}
$$

and

$$
m = 1 - \beta^2 = \frac{R_0^2 + (a-a')^2}{R'^2}\ .
$$

NUMERICAL SET — UP

Backing up to the EFIE which must be solved, we must now consider the approximation forms which will be used in the solution. In this context, we approximate the current I with a set of functions we will call basis functions. For this development, we approximate I by piecewise linear functions. For simplicity, this set of functions is most easily described with linear spline type functions shown as triangular functions.

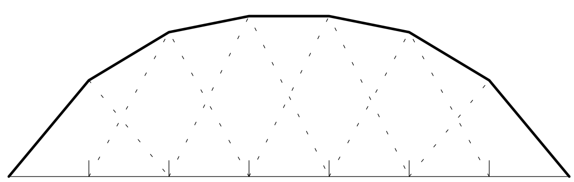

Figure 2. Current expansion on a wire segment.

The EFIE equation requires the differentiation of the current which becomes a series of pulse functions as shown.

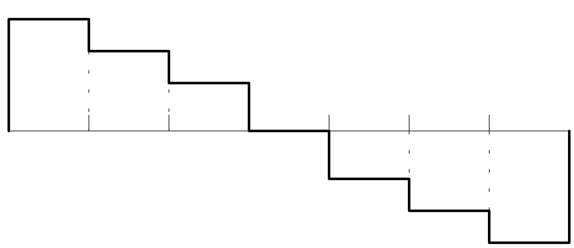

Figure 3. Charge expansion on a wire segment.

With this formalism we may write

$$I(\ell') = \sum_n I_n \, \mathrm{T}_n(\ell') \tag{27}$$

where

$$T_n(\ell') = \left(1 - \frac{\ell' - \ell_n}{\ell_{n-1} - \ell_n}\right) P_{n-1}(\ell') + \left(1 - \frac{\ell' - \ell_n}{\ell_{n+1} - \ell_n}\right) P_{n+1}(\ell') \tag{28}$$

with the P's representing the pulses adjacent to ℓ_n. The corresponding charge is defined in terms of the derivative of the current given by

$$\frac{\partial I}{\partial \ell'} = \sum_n I_n \left[\frac{P_{n-1}(\ell')}{\ell_n - \ell_{n-1}} - \frac{P_{n+1}(\ell')}{\ell_{n+1} - \ell_n}\right]. \tag{29}$$

In applying the method of moments [Harrington, 1968] to solve (10) numerically, we use the above expansion functions. This method also requires a set of testing functions used to force the equation equality using weighted integrals, commonly called a weak equality. Simple pulses extending from the center of each segment to the adjacent segment, denoted by $P_n(\ell)$, are used for testing. This process results in a matrix equation

$$[\mathbb{V}_m] = [\mathbb{Z}_{mn}]\,[I_n]\,, \tag{30}$$

which is solved for the current amplitudes I_n. The $\mathbb{Z}_{mn}$ are evaluated using

$$\mathbb{Z}_{mn} = \frac{1}{\mathrm{j}\omega\epsilon} \int_{\ell_{m-\frac{1}{2}}}^{\ell_{m+\frac{1}{2}}} \hat{\boldsymbol{\ell}} \cdot \left\{ k^2 \int_{\ell_{n-1}}^{\ell_{n+1}} \hat{\boldsymbol{\ell}}' \, T_n(\ell') \, K(\ell,\ell') \, \mathrm{d}\ell' + \nabla \int_{\ell_{n-1}}^{\ell_{n+1}} \hat{\boldsymbol{\ell}}' \, \frac{\partial T_n(\ell')}{\partial \ell'} \, K(\ell,\ell') \, \mathrm{d}\ell' \right\} \mathrm{d}\ell. \tag{31}$$

The second term is straight forward to evaluate. Denoted $\mathbb{Z}^{(2)}_{mn}$, we write

$$\mathbb{Z}^{(2)}_{mn} = \frac{1}{\mathrm{j}\omega\epsilon} \left[-\frac{\Psi_{m+\frac{1}{2},n+\frac{1}{2}}}{\ell_{n+1} - \ell_n} + \frac{\Psi_{m-\frac{1}{2},n+\frac{1}{2}}}{\ell_{n+1} - \ell_n} + \frac{\Psi_{m+\frac{1}{2},n-\frac{1}{2}}}{\ell_n - \ell_{n-1}} - \frac{\Psi_{m-\frac{1}{2},n-\frac{1}{2}}}{\ell_n - \ell_{n-1}} \right] \tag{32}$$

with

$$\Psi_{m+\frac{1}{2},n+\frac{1}{2}} = \int_{\ell_n}^{\ell_{n+1}} K(\ell_{m+\frac{1}{2}}, \ell') \, \mathrm{d}\ell'.$$

The first term may not be evaluated in the same simple form. However, the argument is relatively smooth, enabling us to simply approximate the integral denoted $\mathbb{Z}^{(1)}_{mn}$. Thus

$$\mathbb{Z}^{(1)}_{mn} \simeq \frac{k^2}{\mathrm{j}\omega\epsilon 2} \left\{ (\boldsymbol{r}_{m+1} - \boldsymbol{r}_m) \cdot \int_{\ell_{n-1}}^{\ell_{n+1}} \hat{\boldsymbol{\ell}}' \, T_n(\ell') \, K(\ell_{m+\frac{1}{2}}, \ell') \, \mathrm{d}\,\ell' + (\boldsymbol{r}_m - \boldsymbol{r}_{m-1}) \cdot \int_{\ell_{n-1}}^{\ell_{n+1}} \hat{\boldsymbol{\ell}}' \, T_n(\ell') \, K(\ell_{m-\frac{1}{2}}, \ell') \, \mathrm{d}\,\ell' \right\}. \tag{33}$$

This is actually the kind of process used by the original MiniNec writers to evaluate the integral, but they suggest the approximation to the current is a set of pulses, which are not analytically differentiable (possibly a case of semantics). If for (33) we approximate

$$T_n(\ell') \simeq \frac{1}{2}\left[P_{n-1}(\ell') + P_{n+1}(\ell')\right], \tag{34}$$

we obtain

$$\mathbb{Z}^{(1)}_{mn} \simeq \frac{k^2}{\mathrm{j}\omega\epsilon 4}\Big\{(\boldsymbol{r}_{m+1} - \boldsymbol{r}_m)\cdot\Big[\hat{\boldsymbol{\ell}}_{n+\frac{1}{2}}\Psi_{m+\frac{1}{2},n+\frac{1}{2}} + \hat{\boldsymbol{\ell}}_{n-\frac{1}{2}}\Psi_{m+\frac{1}{2},n-\frac{1}{2}}\Big] + (\boldsymbol{r}_m - \boldsymbol{r}_{m-1})\cdot\Big[\hat{\boldsymbol{\ell}}_{n+\frac{1}{2}}\Psi_{m-\frac{1}{2},n+\frac{1}{2}} + \hat{\boldsymbol{\ell}}_{n-\frac{1}{2}}\Psi_{m-\frac{1}{2},n-\frac{1}{2}}\Big]\Big\}. \tag{35}$$

We sum these two parts to obtain

$$\mathbb{Z}_{mn} = \mathbb{Z}^{(1)}_{mn} + \mathbb{Z}^{(2)}_{mn}. \tag{36}$$

Lumped element loading (including skin effect loss) or input resistance is simply added to the $\mathbb{Z}_{mn}$ of (36) to obtain a modified $\mathbb{Z}_{mn}$ for computation. This treatment of loading follows the development of Poggio [1969].

The evaluation of the source term $\mathbb{V}_m$ may be separated into the cases of source fed antennas and plane-wave incidence. For plane-wave incidence, we simply approximate $\mathbb{V}_m$ by

$$\begin{aligned}\mathbb{V}_m &\simeq -\overline{\boldsymbol{E}}_i(\ell_m)\cdot\frac{(\boldsymbol{r}_{m+1} - \boldsymbol{r}_{m-1})}{2}\\ &\simeq -\overline{\boldsymbol{E}}_{i0}(0)\cdot\frac{(\boldsymbol{r}_{m+1} - \boldsymbol{r}_{m-1})}{2}\,\mathrm{e}^{\mathrm{j}k\boldsymbol{r}_m\cdot\hat{\boldsymbol{r}}_s},\end{aligned} \tag{37}$$

where $\hat{\boldsymbol{r}}_s$ is directed to the plane-wave source. The simplest model to use for a fed wire is a PEC tube with a double ring magnetic source. This model creates a ring of magnetic current on both the inner and outer surfaces of the tube in opposing directions. Thus the magnetic ring sources do not radiate as a result of the cancellation, but contribute to the electric field only at the surface locations of the rings. This provides a modeling foundation for what most people consider an approximate source referred to as the delta gap model. In this case, we simply have $\mathbb{V}_m$ as the applied voltage in the lumped element sense at ℓ_m.

The major concern in this source modeling is the near field effects on computed parameters. A common parameter of interest is the antenna input impedance. A typical numerical experiment performed on the input impedance is to plot convergence as the number of unknowwns is increased. When we model the source by $\mathbb{V}_m$ as a gap model, we cannot actually claim that the numerical process has solved the gap feed problem. The actual source must be related to the basis functions used for the current expansion and related slope discontinuities in the current. It is not possible to establish a precise relationship between the current and source model. However, studies [Mittra and Klien, 1975] have shown that increasing n, the number of unknowns, does not provide convergence of the input impedance. If the source is applied at the center element of a dipole, this convergence does not occur when the impedance is plotted versus $1/n$. (This $1/n$ form of plot is useful in evaluating convergence phenomena versus the number of unknowns.) One possible explanation typically given is round − off error must be the problem. However, increased precision still has the same result. If the source is distributed over several segments so as to maintain the physical dimension of the source, convergence is obtained. This is directly related to the near-field suseptance at the antenna input. This modeling aspect should be kept in mind as the analysis is used for antennas.

The last step in the current analysis for a perfectly conducting wire is the effect of a ground plane, if included. A PEC ground-plane model has been used with the associated image theory. To account for the ground, we simply add to $\mathbb{Z}_{mn}$ an image impedance $\mathbb{Z}^i_{mn}$ given by

$$\mathbb{Z}^i_{mn} = \mathbb{Z}^{(1)i}_{mn} + \mathbb{Z}^{(2)i}_{mn}, \tag{38}$$

where the image impedances are obtained from the same formulae as the original impedances except for the replacment of the following:

$$\ell' \rightarrow \ell^i$$
$$\boldsymbol{r}' \rightarrow \boldsymbol{r}^i = \boldsymbol{r}' - 2\, z'\, \hat{\boldsymbol{z}}$$
$$\hat{\boldsymbol{\ell}} \rightarrow \hat{\boldsymbol{\ell}}^i = -\hat{\boldsymbol{\ell}}' + 2\,\hat{\boldsymbol{z}}\,(\hat{\boldsymbol{z}} \cdot \hat{\boldsymbol{\ell}}').$$

The effects of low loss in the ground plane are typically treated in either an exact manner using the Sommerfeld theory or the simplfied far-field use of the reflection method. A new method which was presented in 1987 [Davis, Sweeney, and Stutzman] is still under study. This latter method is an approximate method which separates the reflected field into two parts, similar to the resultant Sommerfeld form, consisting of a specular part (a standard image) and a perturbed part. The perturbed part is being treated in a reflection method format, while the specular part is treated exactly.

Loading effects are easily added in the manner used by Poggio [1969]. The load at each element is simply added to the diagonal element of the matrix. Likewise, conductor loss may simply be added as a series resistance as would any other load. Such a series resistance would require the evaluation of the resistance per length due to the conductor resistivity and possibly skin-depth.

The program has been used successfully for dipoles, folded dipoles, yagis, loops, and inductively loaded whips. As of yet, no wire antenna structures have failed in the program[2], unless requiring too many unknowns (currently at a maximum of 160 double precision). Sample plots for the current and field pattern of a $\lambda/2$ dipole are shown in Figs. 4 and 5. The program has been written for the CGA display to provide the availablity to the maximum number of users and not limit the number of unknowns by loading the code with extra video driver information.

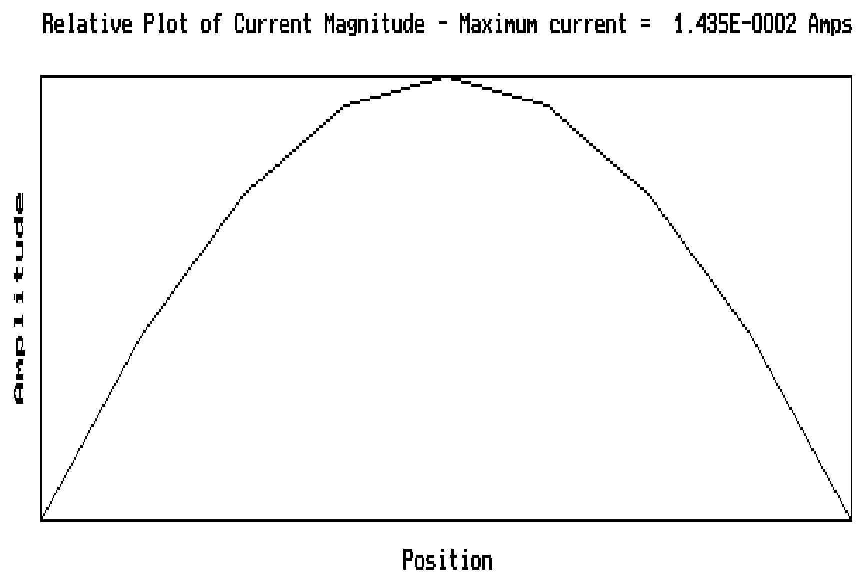

Figure 4. Current plot from the wire program for an 8-segment $\frac{\lambda}{2}$ dipole.

[2]The executable code for and PC is avialable by FTP from gopher.vt.edu under filebox/eng/ee/faculty/WDavis as ANTENNA.EXE, a self-extracting archive.

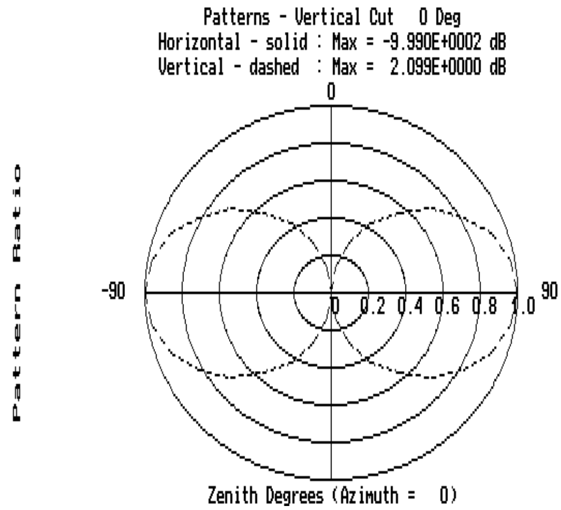

Figure 5. E-plane pattern plot from the wire program for an 8-segment $\frac{\lambda}{2}$ dipole.

The current plot provides the relative amplitude of the current versus position, not versus the unknown number. Thus a problem such as a folded dipole displays a small current section on the end wires, while the remaining current is appropriately distributed along the antenna wires. The formulation presented also has allowed the modeling of coaxial cable types of problems with the associated standing-wave currents. The field pattern is available in ϕ-plane cuts as well as fixed-θ conical cuts. The pattern is displayed for both the ϕ and θ components of the electric field with the signal level relative to an isotropic element, giving the gain for the given pattern.

SUMMARY

This paper has presented the fundamental concepts of wire antenna analysis currently used in a modified version of the program MiniNec. Specific attention has been given both to the current and testing function approximations and the computation of the integral equation kernel. Comments have been made which provide justification for the widely used delta gap model for the antenna source within a tubular model. A form is also presented for which the antenna may be analyzed in a receive mode. The critical areas treated by this paper are the approximations made in both the model and the numerical computation. These approximations include both the circumferential nature of the current and the numerical expansion of the current along the structure. The current expansion is a clarification of the often misled viewpoint that pulse testing and expansion was used, when indeed the current was approximated in a linear manner and the result approximated by pulses for the computation of some integrals. Hopefully this paper provides an approach to the whole problem which is tractable by a wide variety of interested readers.

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NOTE FOR REVIEWER:

DERIVATION OF (R^2)

The derivation of R^2 is non-trivial, but is easily developed based on local coordinates. We choose a local coordinate system with $\widehat{z}$ oriented parallel to $\overline{r}_o$ wire and the orientation such that the relative tilt of the source wire is in the x-z plane. Thus

$$\begin{aligned} \overline{r} &= \overline{r}_o + a(\widehat{x}\cos\phi + \widehat{y}\sin\phi) \\ \overline{r}' &= \overline{r}_s + a'(\widehat{x}\cos\phi'\cos\theta + \widehat{y}\sin\phi' + \widehat{z}\cos\phi'\sin\theta) \end{aligned}$$

to give

$$\overline{r} - \overline{r}' = \overline{r}_o - \overline{r}_s + \widehat{x}(a\cos\phi - a'\cos\phi'\cos\theta) + \widehat{y}(a\sin\phi - a'\sin\phi') - \widehat{z}a'\cos\phi'\sin\theta\ .$$

Now we have

$$\begin{aligned} R^2 = {} & |\overline{r}_o - \overline{r}_s|^2 + a^2 + a'^2 - 2aa'(\cos\phi\cos\phi'\cos\theta + \sin\phi\sin\phi') \\ & + 2(\overline{r}_o - \overline{r}_s)\cdot[\widehat{x}(a\cos\phi - a'\cos\phi'\cos\theta) + \widehat{y}(a\sin\phi - a'\sin\phi') - \widehat{z}a'\cos\phi'\sin\theta] \end{aligned}$$

or

$$\begin{aligned} R^2 = {} & |\overline{r}_o - \overline{r}_s|^2 + \left[a^2 + {a'}^2 - 2aa'\cos(\phi - \phi')\right] \\ & + 2aa'\cos\phi\cos\phi'(1 - \cos\theta) + 2[a\rho'\cos(\phi - \phi_o) + a'\rho\cos(\phi - \phi_s)] \end{aligned}$$

where we need to define ρ, ρ', ϕ_o, and ϕ_s. For ρ' and ρ, let us consider

$$\begin{aligned} \rho' &= (\overline{r}_o - \overline{r}_s)\cdot(\widehat{x}\cos\phi_o + \widehat{y}\sin\phi_o) \\ \rho &= (\overline{r}_o - \overline{r}_s)\cdot(\widehat{x}a'\cos\phi_s'\cos\theta + \widehat{y}a'\sin\phi_s' + \widehat{z}a'\cos\phi_s'\sin\theta) \end{aligned}$$

where ϕ_o and ϕ_s makes ρ' and ρ maximums respectively. Thus,

$$\begin{aligned} (\overline{r}_o - \overline{r}_s)\cdot(\widehat{x}\cos\phi + \widehat{y}\sin\phi) &= \rho'\cos(\phi - \phi_o) \\ (\overline{r}_o - \overline{r}_s)\cdot(\widehat{x}\cos\phi'\cos\theta + \widehat{y}\sin\phi' + \widehat{z}\cos\phi'\sin\theta) &= \rho\cos(\phi' - \phi_s) \end{aligned}$$

Substituting into R^2 we obtain

$$\begin{aligned} R^2 = {} & |\overline{r}_o - \overline{r}_s|^2 + \left[(a + a')^2 - 4aa'\cos^2\left(\frac{\phi - \phi'}{2}\right)\right] \\ & + 4aa'\cos\phi\cos\phi'\sin^2\left(\frac{\theta}{2}\right) + 2[a\rho'\cos(\phi - \phi_o) + a'\rho\cos(\phi' - \phi_s)] \end{aligned}$$